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# Analyzing the effect of noise in the Fitzhugh-Nagumo model of a neuron

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**Abstract-** An analysis of the response of a neuron or an ensemble of neurons represented by non-linear ordinary differential equations without input noise and with stochastic input noise is presented in this paper. The noise introduced in the input for the Fitzhugh-Nagumo system is taken as Gaussian white noise distribution. Simulation of the differential equations is done through numerical methods. Using the method of simulation, we study the interspike interval distribution from the sample paths which have been simulated and compare the results obtained for current inputs both with and without noise.

***Index terms:*** Fitzhugh-Nagumo model, Gaussian white noise, interspike interval, ordinary differential equations, stochastic input, simulation, sample paths.

## 1 INTRODUCTION

The biological plausibility and computational efficiency of a neuronal model is used to determine which model can be best adopted for studying neural encoding and decoding. The transmission of information in the nervous system occurs due to the non-linear transformation of stochastic input currents to discrete spike trains. This transformation is generally noisy. We study the Fitzhugh-Nagumo system and the response of this dynamical system to noise in this paper. This model is a spiking neuron model and obtained by simplifying the Hodgkin-Huxley model according to its phase plane behavior. Fitzhugh used approximate projections of the 4-dimensional Hodgkin-Huxley model and cast it into an approximate 2-dimensional manifold of the original system. This approximated 2-dimensional version of the HH model showed strong similarity with the Van der Pol oscillator. By adding terms to a special case of the Van der Pol oscillator, Richard Fitzhugh obtained the Fitzhugh-Nagumo (FN) model. The two variables that were kept during reduction of the Hodgkin-Huxley model to the FN model are the recovery variable and the excitable variable which are respectively termed as the slow and fast variables. The equations in the FN model control the electrical potential across cell membranes by changing the flow of ionic channels of the cell membrane. As a result there occurs a change in potential which transfers electrical signals from one neuron to another.[1,2] This paper consists of five sections. After a brief introduction in the first section, section 2 deals with the background of the neuronal model in consequence of our study. Mathematical analysis of the model is shown in section 3 while section 4 shows the results obtained through simulation of the model with and without stochastic noise. Finally section 5 deals with conclusion.

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## 2 BACKGROUND

**Hodgkin-Huxley Model**: Alan Hodgkin and Andrew Huxley performed experiments which demonstrated the ion channel model of the action potentials. The experiment was carried out in a giant squid axon and the behavior displayed by the action potential convinced the researchers that there was more than one current active in the axon. The different spiking phenomena exhibited by a neuron exposed to different current simulations are explained by this model. The model represented by a complicated nonlinear ODE system consists of four equations describing the membrane potential, activation of Na current, activation of K current, and inactivation of Na current and is defined by four dimensional vector fields given by the gating variables v, m, n and h respectively [1]. The Hodgkin-Huxley model can finally be formulated by the equation:

$$I_{total} = n^4 \bar{g}_K (V - V_k) + m^3 h \bar{g}_{Na} (V - V_{Na}) + \bar{g}_L (V - V_L) \quad (1)$$

Where, $\bar{g}$ is the maximum conductance of K, Na and the leak current L.

**Fitzhugh-Nagumo model:** The four dimensional Hodgkin-Huxley equations was reduced to two dimensional by Richard Fitzhugh. Fitzhugh noticed that the gating variables v and m changed more rapidly than n during certain time intervals. When the variables h and n were arbitrarily set as constant, a set of two equations could be isolated to describe a two dimensional (v, m) phase plane. This system showed a similarity with the Van der Pol oscillator, which has a non-conservative property and non-linear damping and develops in time according to a second order differential equation. In 1962, an electrical equivalent of the system was proposed by Nagumo. The Fitzhugh-Nagumo (FN) model was used to describe the firing mechanism in an excitable neuron. The FN model obtained by mimicking the nullclines of the HH model is a polynomial reduction model that is formulated by the following equations:

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$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + I(t) \quad (2)$$

$$\frac{dw}{dt} = \mathcal{E}\,(v + a - bw) \quad (3)$$

Where, ‘v’ represents the potential and ‘w’ represents the recovery variable. The parameters a, b, Ɛ are dimensionless, positive and used for the time scale and kinetics of the recovery variable.

Phase plane analysis of the FN model visualizes the temporal evolution of the variables (u, w) in the phase plane. Lines on which a differential is zero in the phase plane are called as nullclines [10]. Analyzing the nullclines of the FN system provides a better understanding of the behavior of the model.

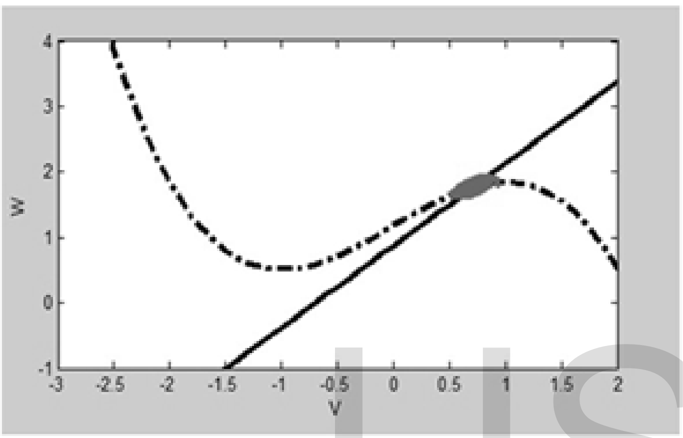


Figure 1: Phase Plan representation of Fitzhugh-Nagumo Model. The V-nullcline (curved line) and the W-nullcline (straight line) intersect at the one fixed points.

## 3 MATHEMATICAL ANALYSIS

For the analysis of a nonlinear system like the FN model, the technique of numerical solution can be applied. We use the Euler Maruyama method for the simulation of the equations of the FN model. This is achieved by discretizing time into small intervals and solving forwards in time to obtain the numerical solution of the dynamical system. The implementation of the equations of the model is done in MATLAB by changing equations (2) and (3) to the form below:

dx(1) = x(1) - (x(1)^3)/3 - x(2) + Iext1 (4)

dx(2) = tau * (x(1) + A - B*x(2)) (5)

To evaluate the Fitzhugh –Nagumo equations and their derivatives, a function ode45 is used to solve the ordinary differential equations.

## 4 SIMULATIONS

Figure 2 shows the dynamics between membrane potential and time. Here membrane potential continuously varies with time. The value of threshold potential is 0.02. Whenever the value of membrane potential reaches the threshold, it suddenly goes to its peak value and generates a spike. After reaching its peak value, it again sets down to its previous value. The dynamics of all spikes generated are same. Parameters used for the simulation are given in Table 1.

Figure 3 shows the change in membrane potential after introducing a stochastic input noise. Noise is introduced to the input parameter of the FN model by using MATLAB’s randn function, which generates random numbers with a Gaussian distribution.

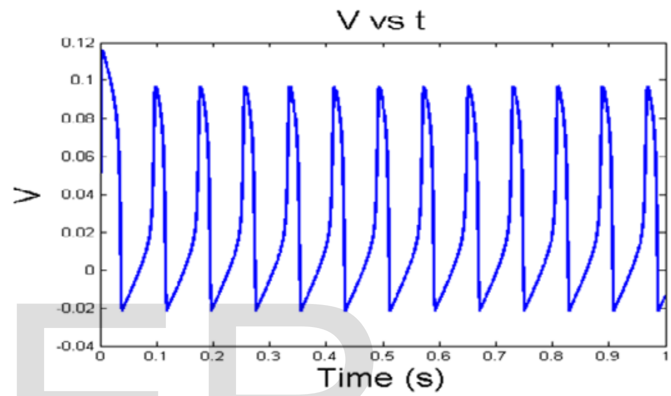


Figure 2: Change in membrane potential in FHN model with respect to time.

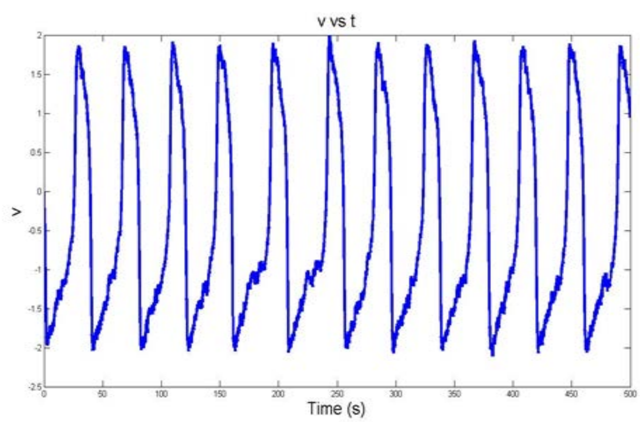


Figure 3: Change in membrane potential in FHN model with respect to time when stochastic noise is introduced.

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Table 1

| Parameters | Value |
|---|---|
| A | 0.7 |
| B | 0.8 |
| $\tau$ | 0.08 |
| $I_{ext}$ | 0.4 |

## 5 CONCLUSION

Fitzhugh-Nagumo systems is used for modeling neuronal behavior as it provides advantage over the Hodgkin-Huxley model in terms of simplicity as fewer parameters are used. The purpose of this model is to capture the qualitative features of the electrical activity along a neuron and hence this model is not meant to be predictive. In this paper we investigated the effect of noise on a FN system. The addition of stochastic input noise to the two coupled nonlinear differential equations generated spikes that were of different nature than those generated without the inclusion of external noise. This model can also be used for testing of stochastic resonance, which is the tendency of some nonlinear systems to exhibit favorable signal response when an optimal amount of noise is added to it. An analytical framework can be used for obtaining more accurate insights into the effect of noise on the stochastic Fitzhugh-Nagumo model.

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